## On the Comte de St. Robert's Method of measuring Heights by means of the Barometer. By William Mathews, Jun., M.A.

(With a Plate of St. Robert's Hypsometrical Sliding Rule.)

'*Barometrical Formula resulting from the Observations made by Mr. James Glaisher.*' Phil. Mag. Feb. 1864.

'*On the Measurement of Heights by the Barometer.*' Phil. Mag. June, 1864.

'*Sur la Mesure des hauteurs à l'aide du baromètre.*' Paris, 1864.

'*Intorno alla Formola barometrica, &c.*' Torino, 1866.

'*Nouvelles Tables hypsomètriques.*' Turin, 1867.

'*Table hypsomètrique pour déterminer rapidement sur place la différence de niveau de deux stations.*' Turin, 1867.

'*De la résolution de certaines Équations à trois variables par le moyen d'une règle glissante.*' Turin, 1867.

'*Tableau graphique donnant à vue l'altitude d'une station au moyen de la seule observation du baromètre et du thermomètre à cette station.*' Turin, 1867.

IN a previous communication to the Alpine Journal (vol. ii. pp. 33–41, 63–67) I gave an explanation of the formula of Laplace, the one generally used in the barometric measurement of heights. I now propose to lay before the members of the Club an account of the remarkable investigations of the Comte Paul de St. Robert of Turin, as contained in the very able memoirs above cited.

Any method whatever that may be employed for the solution of the general problem must depend upon the three following data:—

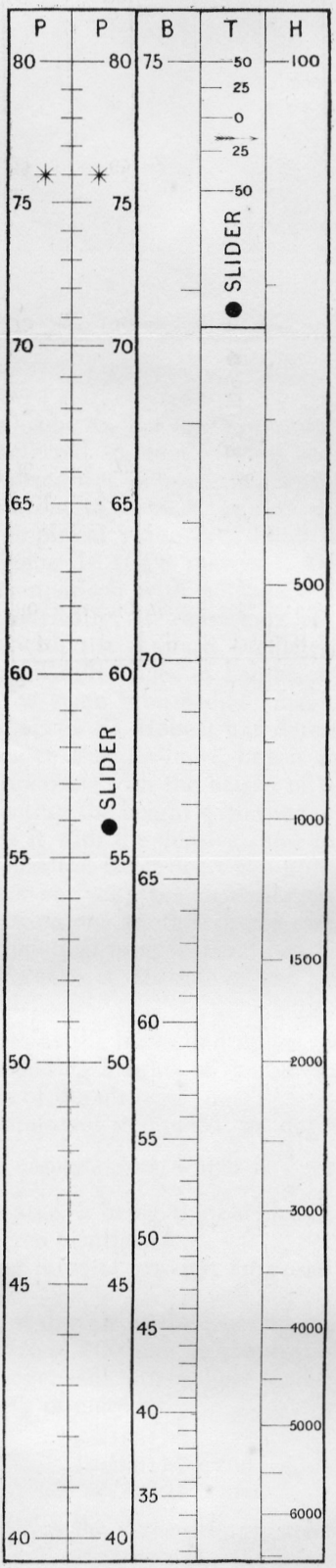
(1) The differential equation connecting the height with the density and pressure.

(2) The relation between the pressure, density, and temperature expressed by the laws of Mariotte and Gay Lussac.

(3) Some other equation, connecting any two or more of these four quantities, founded either on hypothesis or observation, or both.

It is in the choice of this last equation that so much difference has arisen in the manner of treating the problem. Laplace connected the temperature with the height, and assumed a relation between them involving a decrease of temperature in an accelerated ratio in ascending from the surface of the earth. Such a law of decrease was long held by meteorologists; but it would appear, from the recent balloon observations of Mr. Glaisher, that the contrary is the case, and that the ratio is a

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# St-ROBERT'S HYPSOLOGISTA,

*or Hypsometrical Sliding Rule. Natural Size.*

*The Intermediate graduations have been omitted for the sake of clearness.*

*To illustrate Mr Mathews' Paper on the Comte de St Robert's Method of measuring Heights by means of the Barometer.*

retarded one. If this be so, Laplace's formula, as was shown in the paper above referred to, would bring out the results too high. In fact, Laplace was obliged, in order to make theory agree with observation, to reduce the barometric coefficient, and to adopt an empirical value of 18,336 mètres, the true theoretical value being 18,404·8 mètres. As the former was obtained from a comparison with a large number of heights measured trigonometrically, it represents an average value. Laplace's formula with the reduced coefficient will therefore err in defect for small differences of height, and in excess for large ones, while for some intermediate magnitude it will be exactly accurate. M. de St. Robert has determined the latter to be 3,000 feet above the sea-level, but it must not be forgotten that it will increase with the height of the lower station.

Instead of connecting the height with the temperature, M. de St. Robert connects it with the density, and founds his theory on the hypothesis that the latter decreases uniformly in ascending from the earth's surface; the densities at any number of points, equidistant from one another in the same vertical, forming a series in arithmetical progression.

Let $x$ be the difference of altitude of two stations, supposed in the same vertical.

$p_o$, $p$ the pressures at the lower and upper station.

$h_o$, $h$ the corresponding heights of the barometer, reduced.

$\rho_o$, $\rho$ the densities of the air.

$t_o$, $t$ the temperatures expressed in degrees Centigrade, measured from the *absolute zero*, which is $\frac{1}{\alpha}$ or 274° below the freezing point of water, $\alpha$ being the coefficient of dilatation of dry air for one degree Centigrade.

$g$ the accelerating force of gravity, supposed the same at the two stations.

Let the density of dry air at the freezing point of water and a barometric pressure of 760 mm. be the unit of density, H the height of the homogeneous atmosphere under the same conditions, $\sigma$ the density of mercury.

$$\sigma = 10517{\cdot}3$$
$$\mathrm{H} = 10517{\cdot}3 \times 760 \text{ mm.}$$
$$= 7993{\cdot}15 \text{ mètres.}$$

We will now express the three primary relations.

*First*, since the difference between the pressures at any two points of an aerial column, whose transverse section is a unit of area, is equal to the weight of the intervening portion of the column, that is, to the product of its length, mean density, and

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the accelerating force of gravity, we have, denoting the mean density by $\mu$,

$$p_o - p = g\mu x,$$
$$\text{or } x = \frac{1}{g}\ \frac{p_o - p}{\mu}.$$

*Secondly*, from the laws of Mariotte and Gay Lussac, the temperatures being measured from the absolute zero,

$$\rho_o = \frac{1}{a}\ \frac{1}{g\mathrm{H}}\ \frac{p_o}{t_o}. \quad \rho = \frac{1}{a}\ \frac{1}{g\mathrm{H}}\ \frac{p}{t}.$$

*Thirdly*, since, by hypothesis, the density decreases in arithmetical progression,

$$\mu = \frac{\rho_o + \rho}{2}.$$

By the substitution of this value of $\mu$, the first equation becomes

$$x = \frac{2}{g}\ \frac{p_o - p}{\rho_o + \rho}.$$

Or, expressing the densities in terms of the pressures and temperatures,

$$x = 2a\mathrm{H}\ \frac{p_o - p}{\frac{p_o}{t_o} + \frac{p}{t}}$$
$$= 58{\cdot}34\ \frac{p_o - p}{\frac{p_o}{t_o} + \frac{p}{t}}.$$

Replacing the pressures by the lengths of the mercurial columns to which they are proportional, and increasing the coefficient to 58·8 to allow for humidity and diminution of gravity, we have

$$x = 58{\cdot}8\ \frac{h_o - h}{\frac{h_o}{t_o} + \frac{h}{t}} \text{ mètres},$$

which is St. Robert's equation.

This expression for $x$ is of extreme simplicity, is obtained without the aid of the differential calculus, is very easily remembered, and does not involve logarithmic computation. We have next to consider how far it is trustworthy.

Whether or not any proposed law can be accepted as valid throughout the whole extent of the atmosphere may be easily tested by forming three equations, into which the pressure, density, and absolute temperature at the upper stations each enters to the exclusion of the other two. By putting these

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quantities equal to zero we obtain three expressions for the limiting height of the atmosphere, and can observe whether they are consistent with one another, and with the value indicated by the phenomena of twilight and shooting stars, viz. from 40 to 50 miles.*

In the 'Philosophical Magazine' for June, 1864, M. de St. Robert has himself put his formula to this trial, with some rather startling results. But the theory of Laplace, when tested in a similar manner, leads to absurdities scarcely less extravagant; and a law which fails, when pushed to its limit, may nevertheless in practice be very approximately correct. It will therefore be useful to examine the practical working of the new formula for those more moderate elevations with which we are likely, for the present at least, to be concerned, and to compare it under similar circumstances with that of Laplace. For this purpose I propose to calculate by both methods the height of Mont Pourri from the four bases of St. Bernard, Aosta, Geneva, and Turin, from observations made by myself in the year 1862: —

*August* 5, 1862, *Noon.*

| | Bar. reduced and corrected. | Temp. Cent. | Absolute Temp. Cent. |
|---|---|---|---|
| Mont Pourri . . | 484·87 mm. | 3·4 | +274·=277·4 |
| St. Bernard . . | 568 32 | 14·6 | 288·6 |
| Aosta . . . | 707·92 | 27·7 | 301·7 |
| Geneva . . . | 724·53 | 25·8 | 299·8 |
| Turin . . . | 735·89 | 29·1 | 303·1 |

The height of Mont Pourri above St. Bernard, according to the formula of St. Robert, is calculated in the following manner:—

$$h_o = 568{\cdot}32$$
$$h = 484{\cdot}87$$
$$h_o - h = 83{\cdot}45$$
$$58{\cdot}8\,(h_o - h) = 4906{\cdot}860$$
$$\frac{h_o}{t_o} = \frac{568{\cdot}32}{288{\cdot}6} = 1{\cdot}969$$
$$\frac{h}{t} = \frac{484{\cdot}87}{277{\cdot}4} = 1{\cdot}748$$
$$\frac{h_o}{t_o} + \frac{h}{t} = 3{\cdot}717$$
$$x = \frac{4906{\cdot}86}{3{\cdot}717}$$
$$= 1320{\cdot}1 \text{ m.}$$

* I am indebted to the kindness of Sir John Herschel for an exhaustive criticism of St. Robert's formula from this point of view.



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Determining in a similar way the height of the mountain above Aosta, Geneva, and Turin, and making parallel calculations by the tables of Delcros, founded on the formula of Laplace, we obtain the following comparative results:—

| | Height of Lower Station. | By Laplace's Formula. | By St. Robert's Formula. | Difference. |
|---|---|---|---|---|
| St. Bernard . | 2478·3 | 1314·7 | 1220·1 | —5·4 |
| Aosta . . | 600·0 | 3210·7 | 3203·5 | 7·2 |
| Geneva . . | 408·0 | 3395·5 | 3383·4 | 12·1 |
| Turin . . | 285·24 | 3549·2 | 3535·3 | 13·9 |

And for the height of Mont Pourri above the sea-level:—

| | By Laplace's Formula. | By St. Robert's Formula. |
|---|---|---|
| St. Bernard . . | 3793·0 | 3798·4 |
| Aosta . . . | 3810·7 | 3803·5 |
| Geneva . . . | 3803·5 | 3791 4 |
| Turin . . . | 3834·44 | 3820·54 |
| Mean . . . | 3810·41 | 3803·46 |

It is evident from these figures that very considerable reliance may be placed in the formula of St. Robert. It gives, as it ought to do, a mean height somewhat less than the method of Laplace, while the separate results are more closely accordant, the difference between the extreme values being 22·14 m. in the one case as against 41·44 m. in the other.

It is worth remarking that the calculations may be made in English measures just as readily as in metrical. We have only to express $h_o$ and $h$ in inches and decimals, and to replace the coefficient of 58·8 mètres by 193 feet; the temperatures being measured by the Centigrade thermometer.

Notwithstanding the many advantages of the formula, the numerical calculations involved in it are unpleasantly laborious; and, in order to remedy this defect, its author has constructed the elaborate series of tables entitled 'Nouvelles Tables Hypsomètriques.'

If in the original equation

$$x = \frac{2}{g}\,\frac{p_o - p}{\rho_o + \rho}$$

we replace $p_o$, $p$ by their values $g\sigma h_o$, $g\sigma h$, it becomes

$$x = \sigma\,\frac{h_o - h}{\frac{1}{2}(\rho_o + \rho)}$$

$$= 10517{\cdot}3\,\frac{h_o - h}{\frac{1}{2}(\rho_o + \rho)}.$$

The equation in this form is taken as the basis of the tables, the product 10517·3 $(h_o - h)$ being tabulated for every centi-

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mètre. The densities are tabulated in a double entry table for every centimètre from 20 to 80, and every degree Centigrade from $-20°$ to $39°$. There are also tables for the corrections due to humidity, latitude, and decrease of gravity.

I have tried these tables, but I cannot say that I am favourably impressed with them. Troublesome interpolations for the determination of the densities are almost always necessary, and, after all, the value of $x$ cannot be obtained without the actual performance of a division.*

It is not surprising, therefore, that M. de St. Robert should have endeavoured to put his equation into a form susceptible of easier numerical calculation. By regarding his theory from another point of view he has adapted it to a different method of procedure, and has deduced from it approximate results of great simplicity and usefulness.

Since, by hypothesis, the density decreases uniformly, the total decrease of density ($\rho_o - \rho$) due to the height $x$ must be equally distributed over each mètre of elevation. In other words, the decrease of density for each mètre, or the fraction $\frac{\rho_o - \rho}{x}$, must be a fixed fractional part of the density at the lower station. This law may be expressed by the equation

$$\frac{\rho_o - \rho}{x} = \frac{\rho_o}{c};$$

whence

$$\rho = \left(1 - \frac{x}{c}\right)\rho_o,$$

where $c$ is a constant quantity.

Or, replacing $\rho$, $\rho_o$ by the quantities $\frac{h}{t}$, $\frac{h_o}{t_o}$, to which they are proportional,

$$\frac{h}{t} = \left(1 - \frac{x}{c}\right)\frac{h_o}{t_o}.$$

By determining $x$ from the expression

$$x = 58{\cdot}8\,\frac{h_o - h}{\frac{h_o}{t_o} + \frac{h}{t}},$$

* To facilitate the calculations required by his formula, M. de St. Robert recommends the following Arithmetical Tables:—

'*La division réduite à une addition.*' Par Picarte. Paris: Mallet, Bachelier; or,

'*Tables de calcul où se trouvent les multiplications et divisions toutes faites.*' Par Crelle. Berlin, 1864.



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into which, it will be observed, $c$ does not enter, the value of $c$ may be obtained from the previous equation.

The theory does not require that this quantity should be absolutely constant. It is sufficient that it should be constant for any given time, and for such portions of the same vertical as are included within those atmospheric strata to which our observations are likely to extend. In fact, if we endeavour to ascertain the value of $c$ from a series of observations made at different hours and in various localities, we obtain results differing somewhat from one another, and the determination of the limits within which this quantity ranges is still a desideratum. These limits, however, are sufficiently near to enable us to adopt without serious error the average value of 12,500 mètres determined by M. de St. Robert.

The adoption of a fixed numerical value for $c$ has enabled M. de St. Robert to solve approximately a very useful problem which is constantly occurring in practice. Suppose a mountaineer to be stationed at such a place as Zermatt, the altitude of which is either known, or can be calculated from a series of comparisons, at stated hours, with regular observatory observations. On leaving his head-quarters for a day's excursion, he carries with him his barometer and thermometer, and observes them at a number of points the heights of which he wishes to determine. But how is this to be effected without simultaneous observations at Zermatt? As far as the barometer is concerned, this want may be supplied by reading it at starting and returning, and interpolating intermediate values; but the same course cannot be pursued with the temperature. The mean of the morning and evening temperatures, for example, would be very far from that of mid-day.

The problem which presents itself is, therefore, *To determine the difference of level between two stations, when we know the height of the barometer and the temperature of the air at one of them, and the height of the barometer only at the other.*

From the two equations

$$\frac{h}{t} = \left(1 - \frac{x}{c}\right) \frac{h_o}{t_o}.$$

$$x = \mathrm{A} \frac{h_o - h}{\frac{h_o}{t_o} + \frac{h}{t}}, \text{ where } \mathrm{A} = 58{\cdot}8,$$

we can eliminate either of the temperatures.

Substituting in the second the value of $\frac{h_o}{t_o}$ derived from the first, it becomes

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$$x = A\,\frac{h_o - h}{\frac{h}{t}\left(1+\frac{c}{c-x}\right)}$$

$$= A\,\frac{t}{h}(h_o - h)\frac{c-x}{2c-x}$$

$$= 2z\,\frac{c-x}{2c-x}, \text{ where } z = \tfrac{1}{2}A t\left(\frac{h_o}{h}-1\right);$$

whence $$x^2 - 2(c+z)x + 2cz = 0,$$

and $$x = c + z - \sqrt{c^2+z^2},$$

taking the radical with a negative sign, since $x$ must be less than $c$.

For any given value of $h_o$, or, as we may for convenience term it, for any given *Barometric Horizon*, we can, from these expressions, construct a table of double entry, in which the values of $x$ are tabulated for any number of consecutive values of $h$ and $t$. The most convenient horizon is 760 mm., as most nearly coinciding with the sea-level; and M. de St. Robert presents us, on a single octavo page, with a table constructed for this horizon, in which the pressures are tabulated in a vertical column for every centimètre, and the temperatures in a horizontal one at intervals of 5°, the heights being read off at the intersection of the two columns.

As an example of the application of this table, let us calculate the altitude of Mont Pourri, having given,

$$h = 484{\cdot}87 \text{ mm.}; \quad t = 3{\cdot}4° \text{ C.}$$

For 48 cm. and 0° C. we find 3845 m.,
with a diff. of 171 m. for 1 cm., and of 55 m. for 5° C.

| | | |
|---|---|---|
| Add for 3·4° | 110 × ·34 = | 37 |
| | | 3882 |
| Deduct for ·487 cm. | 171 × ·487 = | 83 |
| Altitude of Mont Pourri | = | 3799 m., |

which agrees very nearly with the previous results.

The barometric pressure of 760 mm. may be termed the *Standard Barometric Horizon*, the table being computed from the formulæ,

$$x = c + z - \sqrt{c^2+z^2}$$

$$z = \tfrac{1}{2}A t\left(\frac{760}{h}-1\right).$$

A similar table might of course be computed for any other

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horizon, but happily this is not necessary. It will be observed that the quantities $h_o$, $h$ enter as a ratio only into the equation

$$z = \tfrac{1}{2} A t \left(\frac{h_o}{h} - 1\right).$$

If, therefore, we take a subsidiary barometric height $k$, such that $\frac{760}{k} = \frac{h_o}{h}$, or $k = \frac{760}{h_o} h$, the height of the point ($h$) above ($h_o$) will be the same as that of the point ($k$) above the standard horizon. In other words, it is immaterial whether we enter, with the argument $h$, a table computed to the horizon $h_o$, or, with the argument $k$, a table computed to the horizon of 760 mm. The latter may consequently be made to do duty in every possible case.

By this ingenious but simple artifice M. de St. Robert enables observers to obtain any approximate results by the easiest numerical calculations, and therefore to multiply observations to an almost indefinite extent. Suppose, for instance, that the traveller stationed at Zermatt reads his barometer on starting for a day's excursion, and finds the height to be $h_o$. Dividing 760 by this quantity, he obtains the ratio $m$, which will probably be constant throughout the day. If $h_1$, $h_2$, $h_3$ be the barometric readings at any number of stations whose heights it is wished to determine, he has only to multiply these by the factor $m$ to obtain $k_1$, $k_2$, $k_3$, &c., the corresponding quantities referred to the standard horizon, and, with the latter as arguments, the height of each station above the starting point may be at once found from the table. The mercurial barometer might indeed be left behind at Zermatt, and the day's observations made with a carefully compared aneroid, and it is scarcely necessary to point out how very greatly this method of calculation increases the usefulness of the aneroid barometer.

But M. de St. Robert has not contented himself with simplifying calculation; he has invented an instrument which enables us to dispense with it altogether. He has not yet published any description of it, but he has courteously supplied me with full particulars, and permitted me to print them here.

This instrument, a figure of which is annexed (see Plate), is constructed on the principle of the *sliding rule*, and is called by its inventor the *Hypsologista*. I propose to name it the *Hypsometrical sliding rule*, as somewhat more natural to English ears.

It consists of two distinct sets of scales, engraved, for the sake of convenience, upon the same piece of wood. The first set are the two scales marked P, P, the initial letter of the

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word *Proportion.* These are graduated for every millimètre from 80 to 40 cm., the graduations on the two being exactly similar. One of the scales is fixed, the other moveable.

The second set comprises three scales marked B, T, H, the initial letters of the words *Barometer, Thermometer, Height;* B and H being fixed, and T moveable. The scale B extends from 75 to 32 cm.; it is graduated for every millimètre from 75 to 65 cm., for every two millimètres from 65 to 60 cm., and for every five millimètres from 60 to 33 cm. The scale H extends from 100 to 6500 mètres, the first division being in the same horizontal line with 75 cm. on scale B; it is graduated for every ten mètres from 100 to 1000, and for every fifty mètres from 1000 to 6500. The slider T extends from 50° to −50° C., and is graduated for every 5°; it carries an arrow as an index.

By means of this instrument the following problems can be solved mechanically:—

First Problem.—*Having given the height of the barometer, h, and temperature of the air, t, at any station, to find the altitude of the station above the Standard Horizon.*

Rule.—*Move the slider* T *until the division t coincides with h on the scale* B; *the division of the scale* H *which is opposite to the arrow-head on the slider is the altitude required.*

Second Problem.—*Having given the height of the barometer, h, and the temperature of the air, t, at one station, to find its altitude above another station, where the height of the barometer is* $h_o$.

Rule.—*Move the slider* P *along the fixed scale until the division* $h_o$ *is opposite the division* 76 *cm., which is marked with an asterisk for distinction. The division of the fixed scale which is opposite to h on the slider is the corresponding quantity, k, referred to the Standard Horizon.*

*Next, move the slider* T *until the division t coincides with k on scale* B; *the division of the scale* H *which is opposite to the arrow-head is the altitude required.*

Having described the Hypsologista and the method of using it, I will briefly explain the principle of graduation, premising that, for the sake of convenience, the lengths of the barometric column are supposed to be expressed in centimètres.

The first set of scales, P, P, are employed for the solution of the equation,

$$\frac{76}{k}=\frac{h_o}{h}.$$

These are an ordinary logarithmic sliding rule—that is, a rule

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in which the absolute distances between successive divisions are proportional to the logarithms of the numbers which the divisions indicate.

Putting the equation under the more general form,

$$\frac{k_o}{k}=\frac{h_o}{h},$$

taking the logarithm of both sides, and introducing a linear coefficient C, for the purpose of fixing the absolute distances between consecutive divisions of the rule, we have

$$C\,(\log.\,k_o-\log.\,k)=C\,(\log.\,h_o-\log.\,h).$$

The two members of this equation being precisely similar, the graduation of the limb and slider must be identical; and if X denote the absolute distance of the division ($h$) from that of ($h_o$),

$$X=C\,(\log.\,h_o-\log.\,h).$$

The value of C is determined by giving to the whole rule a certain fixed length. In order to make the instrument conveniently portable, M. de St. Robert has fixed upon two décimètres as the distance between the extreme divisions of the rule, and as it is graduated from 80 to 40 centimètres,

$$200\text{ mm.}=C\,(\log.\,80-\log.\,40)$$
$$=C\log.\,2\,;$$

whence $$C=664{\cdot}39\text{ mm.}$$

The absolute distance in millimètres of any division ($h$) measured from the first division (80) is therefore given by the equation,

$$X=664{\cdot}39\,(\log.\,80-\log.\,h)$$

The limb and slider are both graduated from this formula by writing for $h$ successively each of the natural numbers from 79 to 40, and the spaces between consecutive graduations are then divided into ten equal parts.

The second set of scales, B, T, H, are employed for the solution of the equation,

$$\frac{x(2c-x)}{A(c-x)}=(274+t)\left(\frac{76}{h}-1\right).$$

In the memoir entitled *De la résolution de certaines Equations à trois variables par le moyen d'une règle glissante*, M. de St. Robert has pointed out that an equation between three variables can always be solved by means of a sliding rule, when it can be put into the form,

$$X=Y+Z,$$

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where X, Y, Z are respectively functions of each of the variables only.

By taking the logarithm of each side of the above equation, and introducing a linear coefficient C, it becomes

$$C \log. \frac{x(2c-x)}{A(c-x)} = C \log. (274+t) + C \log. \left(\frac{76}{h}-1\right).$$

And putting

$$X = C \log. \frac{x(2c-x)}{A(c-x)}$$

$$Y = C \log. (274+t)$$

$$Z = C \log. \left(\frac{76}{h}-1\right),$$

we have

$$X = Y + Z,$$

and the condition is therefore fulfilled.

The scales B, T, and H are graduated according to the above expressions for Z, Y, X; C being taken at 100 mm., which gives to the scales B and H about the same length as P.

By making X, Y, and Z successively equal to 0, we get

$$x = \frac{A}{2} \text{ or } 29{\cdot}4, \text{ very nearly;}$$

$$t = -273$$

$$h = 38.$$

The values of X, Y, and Z corresponding to the initial division of each scale may be denoted by $X_o$, $Y_o$, $Z_o$; and it is preferable to commence the graduation at the initial divisions rather than at the points where X, Y, and Z vanish.

The scale H cannot be commenced at 0 m., because that value of $x$ makes X infinite. It has therefore been commenced at 100 m., and the scale B at 75 cm.

The absolute distance in millimètres, on the scale B, of the division ($h$), measured from the point (75 cm.), is given by the expression $-(Z_o - Z)$ or $Z - Z_o$.

$$Z = 100 \log. \left(\frac{76}{h}-1\right),$$

$$Z_o = 100 \log. \left(\frac{76}{75}-1\right),$$

$$Z - Z_o = 100 \log. 75 \left(\frac{76}{h}-1\right),$$

which is the formula for the graduation of the scale B.

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The absolute distance of the division ($x$) measured from the point (100 m.) is given by the expression $X - X_0$.

$$X = 100 \log. \frac{x(2c-x)}{A(c-x)}$$

$$X_0 = 100 \log. \frac{100(2c-100)}{A(c-100)}$$

$$X - X_0 = 100 \left\{ \log. \frac{c-100}{100(2c-100)} \cdot \frac{x(2c-x)}{c-x} \right\},$$

which is the formula for the graduation of the scale H.

The absolute distance of the division ($t$), measured from the point ($+50°$), is given by the expression $Y_0 - Y$.

$$Y = 100 \log. (274+t)$$

$$Y_0 = 100 \log. (274+50)$$

$$Y_0 - Y = 100 \log. \frac{274+50}{274+t},$$

which is the formula for the graduation of the scale T.

The position of the arrow is determined in the following manner:—

Since the graduations 75 cm. and 100 m. are in the same horizontal line, the arrow must coincide with that value of $t$ which results from writing those values of $h$ and $x$ in the equation,

$$\frac{x(2c-x)}{A(c-x)} = (274+t)\left(\frac{76}{h} - 1\right);$$

whence $t = -17{\cdot}87°$.

If in the formula for the scale T we substitute this value, we get 10·21 mm.; and if we make $t=0$, we get 7·28 mm. The arrow must therefore be placed at an absolute distance of 2·93 mm. from 0°, on the side of the negative temperatures.

I have, in conclusion, to record my obligation to their eminent author for copies of the various memoirs referred to in this paper, and also for numerous letters in explanation of points of detail in which I was at fault. I am happy to add that Mr. J. Hicks, the well-known optician, of No. 8 Hatton Garden, has undertaken the manufacture of the Hypsologista, and is now ready to supply it.

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